Branches in AC circuit

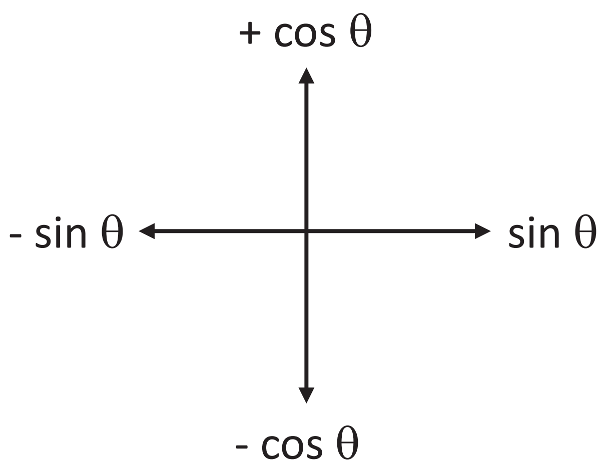
Basic branches

R-branch : Purely resistive branch, i.e. a circuit consists of only 1 or more than 1 resistors
L-branch : Purely inductive branch, i.e. a circuit consists of only 1 or more than 1 inductors
C-branch : Purely capacitive branch, i.e. a circuit consists of only 1 or more than 1 capacitors

Combined branches

R-L branch : A circuit consists of resistors and inductors
R-C branch : A circuit consists of resistors and capacitors
R-L-C branch : A circuit consists of resistors, inductors and capacitors

Need the following graphical relationships between sine and cosine



$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\cos\theta = \sin(\theta + 90^{o})$

$\sin\theta = \cos(\theta - 90^{o})$

$-\cos\theta = \sin(\theta - 90^{o}) = \sin(\theta + 270^{o})$

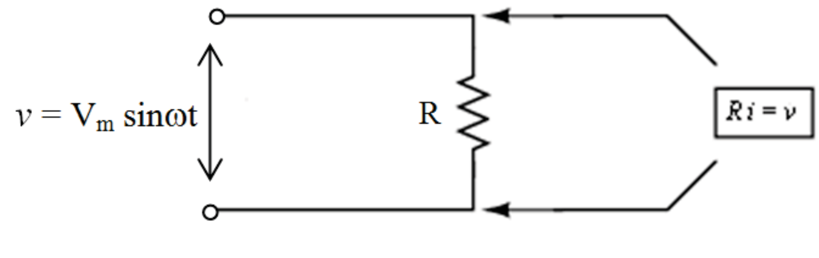
$-\sin\theta = \sin(\theta \pm 180^{o}) = \cos(\theta + 90^{o})$

R-Branch

$$v = V_m \sin\omega t = Ri$$

$$i = \frac{V_m}{R}\sin\omega t$$

$$= I_m \sin\omega t \qquad \left[\frac{V_m}{R} = I_m\right]$$



Impedence, $Z = \frac{\text{Voltage}}{\text{Current}} \angle \theta_v - \theta_i$

Impedence include the angle also.

$Z = X \angle \theta$

$Z_R = X_R \angle \theta_R$

θ_v = Phase of v

θ_i = Phase of i

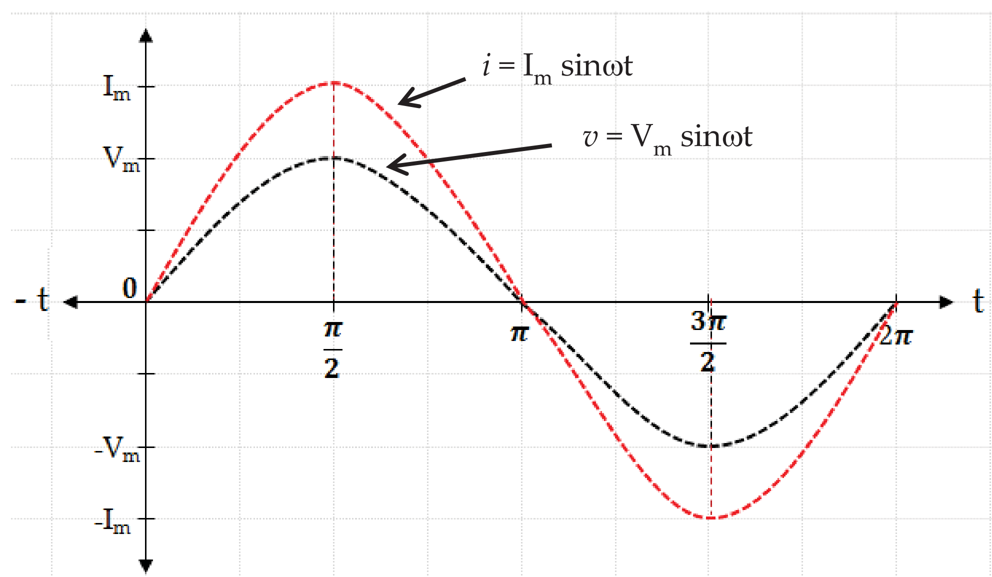
$X_R = \frac{V_m}{I_m} \qquad \theta_R = \theta_v - \theta_i$

$$X_R = \frac{V_m}{I_m} = \frac{V_m}{\frac{V_m}{R}} = R$$

$$\theta_R = \theta_v - \theta_i = 0^\circ - 0^\circ = 0^\circ$$

$$\therefore Z_R = R \angle 0^\circ$$

$\therefore$ Phase difference between Signals of v and i is 0° or inphase.



L-Branch

$$v = V_m \sin\omega t = L\frac{di}{dt}$$

$$v = V_m \sin\omega t = L\frac{di}{dt}$$

$$\therefore\ L\frac{di}{dt} = V_m \sin\omega t$$

or, $$di = \frac{V_m}{L}\sin\omega t\, dt$$

Integrating both side,

$$\int di = \int \frac{V_m}{L}\sin\omega t\, dt$$

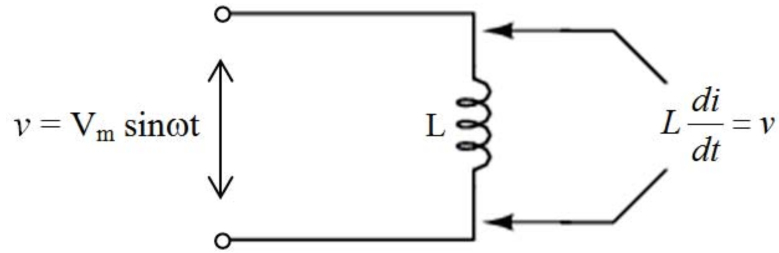
or, $$i = \frac{V_m}{L}\int \sin\omega t\, dt$$

$$= \frac{V_m}{L}\left\{(-\cos\omega t)\cdot\frac{1}{\omega}\right\}$$

$$= -\frac{V_m}{L\omega}\cos\omega t$$

$$= \frac{V_m}{\omega L}\sin(\omega t - 90^\circ)$$

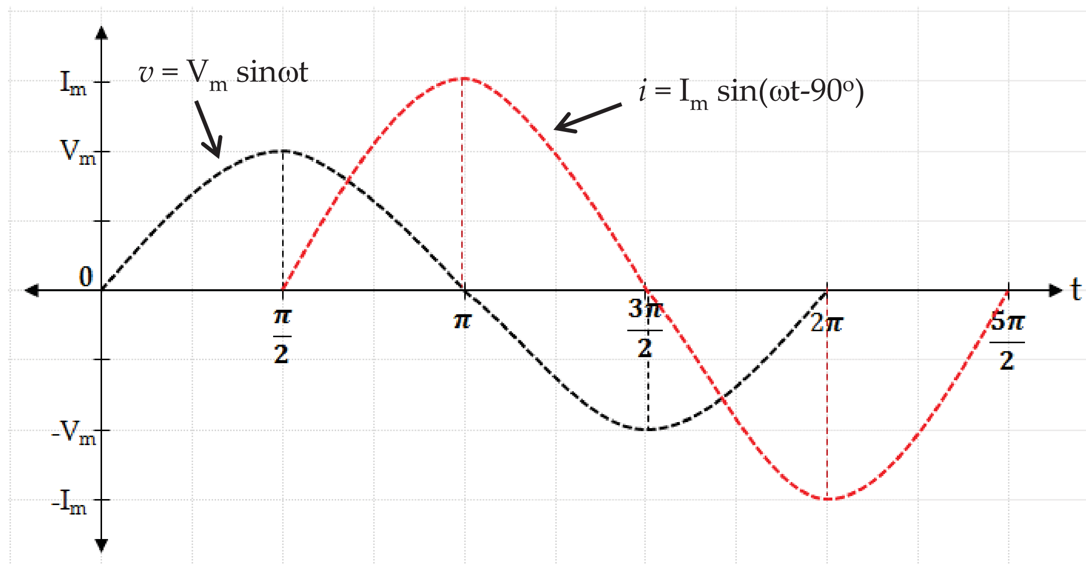
or, $$i = I_m \sin(\omega t - 90^\circ)$$



$$\therefore\ Z_L = X_L \angle\theta_L = \frac{V_m}{I_m}\angle\theta_v - \theta_i$$

$$= \frac{V_m}{\frac{V_m}{\omega L}}\angle 0^\circ - (-90^\circ) = \omega L \angle 90^\circ$$

that is current lags voltage by 90°.



C-Branch

$$v = V_m \sin\omega t = \frac{q}{C}$$

$$\therefore \frac{q}{C} = V_m \sin\omega t$$

or, $q = C V_m \sin\omega t$

Differentiating d both side with respect to t,

$$\frac{dq}{dt} = \frac{d}{dt}(C V_m \sin\omega t)$$

or, $i = C V_m \frac{d}{dt}(\sin\omega t)$

or, $i = C V_m \cos\omega t \cdot \omega$

$= \omega C V_m \cos\omega t$

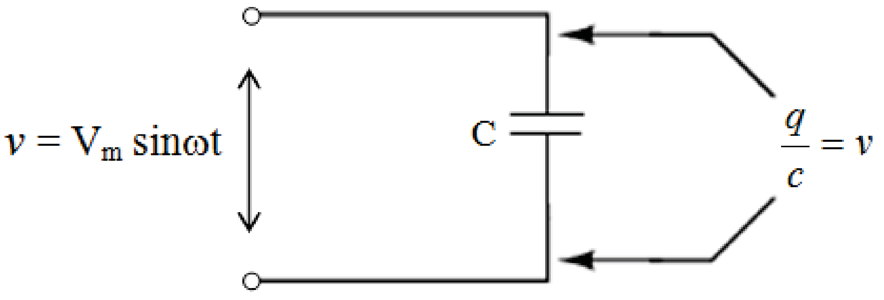
$= \omega C V_m \sin(\omega t + 90^\circ)$

or, $i = I_m \sin(\omega t + 90^\circ)$

$$Z_c = X_c \angle \theta_c = \frac{V_m}{I_m} \angle \theta_v - \theta_i$$

$$= \frac{V_m}{\omega C V_m} \angle 0^\circ - 90^\circ$$

$$= \frac{1}{\omega C} \angle -90^\circ$$



That is current leads voltage by 90°.